PT2

HALF YEARLY EXAMINATION 2022-23

MATHEMATICS

CLASS XII **Time : 3 hrs.**
Mark : 80

General Instructions:

Question paper is divided in the Five sections, Section A - has 18 MCQ's and 2 Assertion-Reason based questions of 1 mark each. Section B has 5 Very Short Answer type 2 marks. Section C - 6 Short Answer type 3 marks, Section D - 4 Long Answer type 5 marks, Section E-3 Source based /case based/passage based/integrated units of assessment. (4 Marks each)

SECTION - A

(Multiple Choice Questions. Each question carries 1 mark)

1. The maximum number of equivalence relations on the set A = {1, 2, 3} is

a) 5 b) 1 c) 3 d) 2

2. If the function $f:\overrightarrow{R}-\{1,-1\}\to$ A defined by $f(x)=\frac{x^2}{1-x^2}$, is surjective, then A is equal to

a) R – [-1, 0) b) R – {-1} c) [0, ∞) d) R – (-1,0)

3. Let (x) = $\sqrt{\cos x}$ Then, dom (f) = ?

a) $\left[\frac{3\pi}{2}, 2\pi\right]$ b) $\left[0, \frac{\pi}{2}\right]$ c) $\left[0, \frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2}, 2\pi\right]$ d) None of these

4. Let matrix A of order 3 is such that $A^2 = 2A - I$ where I is an identity matrix of order 3. Then for $n \in N$ *and* $n \geq 2$, A^n is equal to :

a) $nA - I$ b) $2^{n-1}A - (n-1)I$ c) $nA - (n-1)I$ d) $2^{n-1}A - I$

5. Let A be set of 4 × 4 skew-symmetric matrices whose entries are –1, 0 or 1. If there are exactly four 0's, six 1's and six -1 s then number of such matrices in set A is equal to :

a) 729 b) 32 c) 64 d) 243

6. If A is a square matix then (A + A) is

a) A skew-symmetric matrix b) A symmetric matrix
c) A null matrix d) An identity matrix

7. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to :

a) I b) $I - A$ c) $I + A$ d) 0

8. If the function $f(x)=\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$ is continuous at each point of its domain, then the value of f(0) is

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 2 d) $-\frac{1}{3}$

9. If $y = \sqrt{\sin x + y}$ *then* $\frac{dy}{dx}$ is equal to

a) $\frac{\cos x}{2y-1}$ b) $\frac{\sin x}{1-2y}$ c) $\frac{\cos x}{1-2y}$ d) $\frac{\sin x}{2y-1}$

10 If y = $x^{x^{\cdot^{\cdot^{\infty}}}}$, then x(1 – y logx) $\frac{dy}{dx}$ is equal to

a) x^2 b) y^2 c) xy^2 d) x^2y

11. The function f(x) = x^x decreases on the interval

a) (0, e) b) (0, 1) c) $\left(\frac{1}{e}, e\right)$ d) $\left(0, \frac{1}{e}\right)$

12. The coordinates of the point on the curve y = x^2 + 7x + 2 which is closest to the line y = 3x – 3, are :

a) (-2, -8) b) (-8, -2) c) (-3, -10) d) (-10, -3)

13. $\int \frac{\sqrt{\tan x}}{\sin x . \cos x} dx = ?$

a) $2\sqrt{\sec x} + C$ b) $2\sqrt{\tan x} + C$ c) $-2\sqrt{\tan x} + C$ d) None of these

14. If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, *then* $\frac{dy}{dx}$ is equal to

a) 1 + y^2 b) 1 – y^2 c) y^2 + 1 d) None of these

15. $\int \frac{1}{(2\sin x + 3\cos x)^2} dx$ equals

a) $\frac{2}{2\tan x + 3} + C$ b) $-\frac{1}{2(3+2cosx)} + C$

c) $-\frac{1}{2(2\tan x + 3)} + C$ d) $\frac{2}{3+2\sin x} + C$

16. The area bounded by the curve y = x|x|, the x – axis and the ordinates x = -1 and x = 1 is given by

a) 0 b) $\frac{2}{3}$ c) $\frac{1}{2}$ d) None of these

17. The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3 x . \sin^2 3x$ (2 $\sec^2$ x.$\sin^2$ 3x + 3 tan x.sin 6x) dx is equal to :

a) $-\frac{1}{18}$ b) $\frac{9}{2}$ c) $\frac{7}{18}$ d) $-\frac{1}{9}$

18. Let a function f defined from $R \rightarrow R \; as \; f(x) = \begin{cases} m-x & for \quad x \le 1 \\ 2mx+1 & for \quad x > 1 \end{cases}$ if the function is onto on R, then the range of m is :

a) [-2, 0) b) $[-2, \infty)$ c) $(0, \infty)$ d) {-2}

Assertion Reason based question.

In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

19. Assertion (A) : The value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} \; is \pm 2\sqrt{2}$

Reason (R) : The determinant of a matrix A order 2 × 2, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is ad – bc.

20. Assertion (A) : *f*(x) = logx is defined for all $x \in (0, \infty)$

Reason (R) : If *f* '(x) > 0, then *f*(x) is strictly increasing function.

SECTION - B (5 × 2 = 10 Marks)

21. Show that the function f : $\rightarrow$ N given by f (1) = f (2) = 1 and f (x) = x - 1 for every x $\geq$ 2 , is onto but not one - one.

22. If $x = \sqrt{a^{\sin^{-1}t}}, \; y = \sqrt{a^{\cos^{-1}t}},$ show that $\frac{dy}{dx} = \frac{-y}{x}$

23. Evaluate : $\int \frac{x+2}{(x+1)} dx$

OR

Evaluate : $\int \frac{18}{(x+2)(x^2+4)} dx$

24. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2).$

OR

Write the value of $\tan^{-1}\left|2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right|.$

25. Find $\int \frac{2\cos x}{(1-\sin x)\left(1+\sin^2 x\right)} dx$

SECTION - C (6 × 3 = 18 Marks)

26. If y = $(x)^x + (\sin x)^x$, then find $\frac{dy}{dx}$.

27. Evaluate the definite integral $\int_0^{\pi} \frac{1}{1+\sin x} dx$

OR

Evaluate: $\int \sin^3 \sqrt{x} dx$

28. Evaluate: $\int \frac{4x^4 + 3}{(x^2+2)(x^2+3)(x^2+4)} dx$

29. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} x^{10} - 1, & if\ x \le 1 \\ x^2, & if\ x > 1 \end{cases}$

30. Show that y = log (1 + x) – $\frac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.

OR

Find the absolute maximum value and the absolute minimum value for the function $f(x) = 4x - \frac{1}{2}x^2$, in the given interval $x \in \left|-2, \frac{9}{2}\right|$.

31. Using method of integration find the area bounded by the curve | x | + | y | = 1.

OR

Find the area of the region bounded by the parabola y = x^2 and y = |x|.

SECTION - D (4 × 5 = 20 Marks)

32. Let L be the set of all lines in xy plane and R be the relation in L define as

R = {(L_1, L_2): $L_1 \parallel L_2$}

Show that R is an equivalence relation.

Find the set of all lines related to the line y = 2x + 4.

33. A square tank of capacity 250 cubic meters has to be dug out. The cost of the land is ₹50 per square metre. The cost of digging increases with the depth and for the whole tank, it is ₹(400 × h^2) where h metres is the depth of the tank. What should be the dimensions of the tank so that the cost is minimum?

OR

Find the values of x for which the function f(x) = $[x(x-2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to the x-axis.

34. Using integration, find the area of the triangle formed by positive X-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $\left(1, \sqrt{3}\right)$.

OR

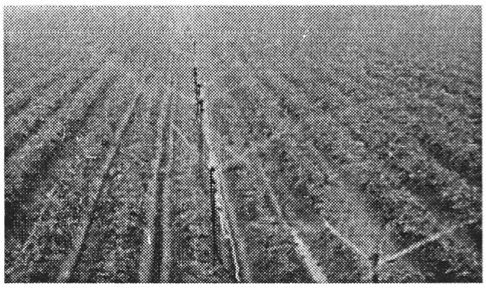
Using the method of integration find the area of the $\Delta ABC,$ coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3).

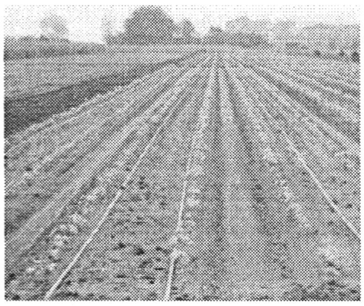
35. Use product $\begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix} \begin{vmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{vmatrix}$ to solve the system of equations $x + 3z = 90$, $-x + 2y - 2x = 4$, $2x - 3y + 4z = -3$.

SECTION - E COMPETENCY BASED

36. Two farmers Shyam and Balwan Singh cultivate only three varieties of pulses namely Urad, Masoor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.

September sales (in ₹)





$$A = \begin{array}{c} \quad \begin{array}{ccc} \textit{Urad} & \textit{Masoor} & \textit{Mung} \end{array} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{c} \\ \textit{Shyam} \\ \textit{Balwan Singh} \end{array}$$

October sales (in ₹)

$$B = \begin{array}{c} \quad \begin{array}{ccc} \textit{Urad} & \textit{Masoor} & \textit{Mung} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{c} \\ \textit{Shyam} \\ \textit{Balwan Singh} \end{array}$$

Using algebra of matrices, answer the following questions.

i) The combined sales of Masoor in September and October, for farmer Balwan Singh. **1**

ii) Find the combined sales of Urad in September and October, for farmer Shyam also find his decrease in sales of Mung from September to October. **2**

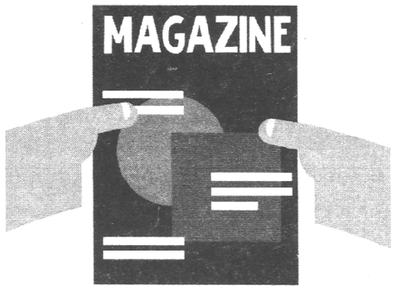
OR

If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety in October.

iii) Which variety of pulse has the highest selling value in the month of September for the farmer Balwan Singh. **1**

37. **Read the case study carefully and answer the questions that follow:**

A magazine company in a town has 5000 subscribers on its list and collects fix charges of ₹3,000 per year from each subscriber. The company proposes to increase the annual charges and it is believed that for every increase of ₹1, one subscriber will discontinue service.



Based on the above information, answer the following questions.

i) If x denote the amount of increase in annual charges then represent revenue R as a function of x also find the value of R if magazine company increases ₹500 as annual charges. **2**

OR

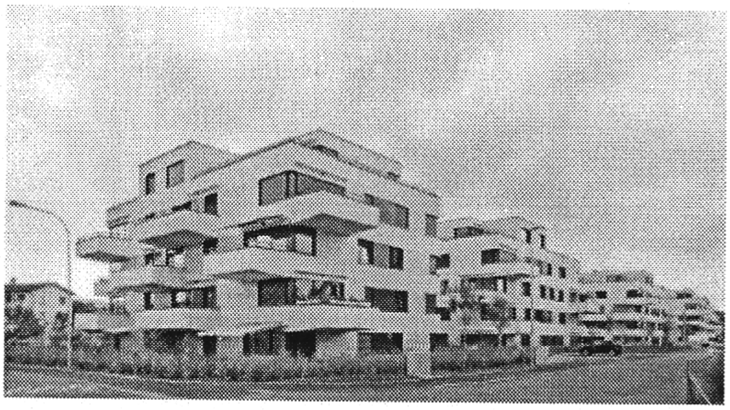
If the revenue collected by the magazine company is ₹1,56,40,000, then find the value of amount increased as annual charges for each subscriber.

ii) What amount of increase in annual charges will bring maximum revenue? **1**

iii) Find the maximum value of revenue. **1**

38. A real estate company is going to build a new residential complex. The land they have purchased can hold at most 4500 apartments. Also, if they make x apartments, then the monthly maintenance cost for the whole complex would be as follows:

Fixed cost = ₹50,00,000. Variable cost = ₹$(160x - 0.04x^2)$.



Based on the above information, answer the following questions.

i) What will be maintenance cost as a function of x? Also find the value of x at which the function attains its maximum value. **2**

ii) Find the number of apartments, that the complex should have in order to minimize the maintenance cost and find the maintenance cost for each apartment. **2**